Preface

This text is an outgrowth of lectures given at the University of Windsor, Canada. One of our main objectives is *updating* the undergraduate analysis as a rigorous postcalculus course. While such excellent books as Dieudonné's *Foundations of Modern Analysis* are addressed mainly to graduate students, we try to simplify the modern Bourbaki approach to make it accessible to sufficiently advanced undergraduates. (See, for example, §4 of Chapter 5.)

On the other hand, we endeavor not to lose contact with classical texts, still widely in use. Thus, unlike Dieudonné, we retain the classical notion of a derivative as a *number* (or vector), not a linear transformation. Linear maps are reserved for later (Volume II) to give a modern version of *differentials*. Nor do we downgrade the classical mean-value theorems (see Chapter 5, §2) or Riemann–Stieltjes integration, but we treat the latter rigorously in Volume II, inside Lebesgue theory. First, however, we present the modern Bourbaki theory of *antidifferentiation* (Chapter 5, §5 ff.), adapted to an undergraduate course.

Metric spaces (Chapter 3, §11 ff.) are introduced cautiously, after the n-space E^n, with simple diagrams in E^2 (rather than E^3), and many "advanced calculus"-type exercises, along with only a few topological ideas. With some adjustments, the instructor may even limit all to E^n or E^2 (but not just to the real line, E^1), postponing metric theory to Volume II. *We do not hesitate to deviate from tradition if this simplifies cumbersome formulations*, upalatable to undergraduates. Thus we found useful some *consistent*, *though not very usual*, *conventions* (see Chapter 5, §1 and the end of Chapter 4, §4), and an *early use of quantifiers* (Chapter 1, §1–3), even in formulating theorems. Contrary to some existing prejudices, quantifiers are easily grasped by students after some exercise, and help clarify all essentials.

Several years' class testing led us to the following conclusions:

(1) Volume I can be (and *was*) taught even to sophomores, though they only gradually learn to *read* and *state* rigorous arguments. A sophomore often does not even know how to *start* a proof. The main stumbling block remains the ε, δ-procedure. As a remedy, we provide most exercises with explicit hints, sometimes with almost complete solutions, leaving only tiny "whys" to be answered.

(2) Motivations are good if they are brief and avoid terms not yet known. Diagrams are good if they are *simple* and appeal to intuition.

(3) Flexibility is a must. One must adapt the course to the level of the class. "Starred" sections are best deferred. (Continuity is not affected.)

(4) "Colloquial" language fails here. We try to keep the exposition rigorous and *increasingly concise*, but readable.

(5) It is advisable to make the students *preread* each topic and prepare questions in advance, to be answered *in the context* of the next lecture.

(6) Some topological ideas (such as compactness in terms of open coverings) are hard on the students. Trial and error led us to emphasize the sequential approach instead (Chapter 4, §6). "Coverings" are treated in Chapter 4, §7 ("starred").

(7) To students unfamiliar with elements of set theory we recommend our *Basic Concepts of Mathematics* for supplementary reading. (At Windsor, this text was used for a preparatory first-year one-semester course.) The first two chapters and the first ten sections of Chapter 3 of the present text are actually summaries of the corresponding topics of the author's *Basic Concepts of Mathematics*, to which we also relegate such topics as the construction of the real number system, etc.

For many valuable suggestions and corrections we are indebted to H. Atkinson, F. Lemire, and T. Traynor. Thanks!

Publisher's Notes

Chapters 1 and 2 and §§1–10 of Chapter 3 in the present work are summaries and extracts from the author's *Basic Concepts of Mathematics*, also published by the Trillia Group. These sections are numbered according to their appearance in the first book.

Several annotations are used throughout this book:

* This symbol marks material that can be omitted at first reading.

⇒ This symbol marks exercises that are of particular importance.