**Mathematical Analysis I**

by

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*Errata*

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This list of errata records substantive changes made since the version of May 20, 2004.

Version of January 20, 2025

(1) Chapter 5, §9, Problem 7, change

$$\int_0^x \frac{\ln(1-t)}{t}\,dt = \sum_{n=1}^{\infty} \frac{x^n}{n^2} \quad \text{for } x \in [-1,\, 1]$$

to

$$\int_0^x \frac{\ln(1-t)}{t}\,dt = -\sum_{n=1}^{\infty} \frac{x^n}{n^2} \quad \text{for } x \in [-1,\, 1].$$

Thanks to Tim Fierens for this change.

Version of May 18, 2017

(1) Chapter 3, §13, Theorem 2, "star" the statement "(* similarly in $C^n$ and other normed spaces)". Similarly in later sections, "star" statements related to $C^n$.

(2) Chapter 3, §13, Problem 8, change "Prove that" to "Prove that for all sets $A$ and $B$ in $(S, \rho)$ and each $p \in S$".

(3) Chapter 4, §2, beginning of Part II, change

$$(g \circ f)(x) = g(f(x)), \quad s \in S.$$

to

$$(g \circ f)(x) = g(f(x)), \quad x \in S.$$

(4) Chapter 4, §9, Problem 6, change

$$f(x) = \sum_{k=0}^{n} a_k x^k = 0 \quad (n = 2m-1),$$

to

$$f(x) = \sum_{k=0}^{n} a_k x^k = 0 \quad (n = 2m-1,\ a_n \neq 0),$$

(5) Chapter 4, §12, Problem 7, change "Prove that if the functions $f_n$ are constant on $B$, or if $B$ is finite, then a pointwise limit of the $f_n$ on $B$ is also uniform" to "Prove that if each of the functions $f_n$ is constant on $B$, or if $B$ is finite, then a pointwise limit of the $f_n$ on $B$ is also a uniform limit".

(6) Chapter 4, §13, proof of Theorem 1, formula (1), change

$$\epsilon > \sum_{k=m}^{n} |f_k(x)|$$

to

$$\epsilon > \sum_{i=m}^{n} |f_i(x)|$$

and

$$\left|\sum_{k=m}^{n} f_k(x)\right| \quad \text{(triangle law)}.$$

to

$$\left|\sum_{i=m}^{n} f_i(x)\right| \quad \text{(triangle law)}.$$

(7) Chapter 4, §13, proof of Theorem 6, change

$$|x_0 - p| < r \quad \left(r = \frac{1}{\overline{\lim}\ \sqrt[n]{|a|}} = \frac{1}{d}\right),$$

to

$$|x_0 - p| < r \quad \left(r = \frac{1}{\overline{\lim}\ \sqrt[n]{|a_n|}} = \frac{1}{d}\right),$$

(8) Chapter 4, §13, Problem 4(iii) change "$a_n = \dfrac{(-1)^n}{n^p}(\sqrt{n+1} - \sqrt{n})$" to "$a_n = \dfrac{(-1)^n}{n^p}(\sqrt{n+1} - \sqrt{n})$, $p \in E^1$".

(9) Chapter 4, §13, Problem 10, remove "[Hint: Use Problem 9′.]".

(10) Chapter 5, §1, Problem 7, change

$$f(x) = \frac{1}{\ln|x|},\ f(0) = 0.$$

to

$$f(x) = \frac{1}{\ln|x|},\ f(0) = 0,\ p = 0.$$

(11) Chapter 5, §1, Problem 9, change

$$f'(p) = \lim_{\substack{x\to p^+ \\ y\to p^-}} \frac{f(x) - f(y)}{x - y} \neq \pm\infty;$$

to

$$\lim_{\substack{x\to p^+ \\ y\to p^-}} \frac{f(x) - f(y)}{x - y} \text{ exists, is finite, and equals } f'(p);$$

Also, change "Disprove the converse by *redefining* $f$ *at* $p$" to "Show, by *redefining* $f$ *at* $p$, that even if the limit exists, $f$ may not be differentiable".

Version of September 24, 2014

(1) Indent the beginning of Chapter 3, §12.
(2) Change right single-quote to prime in property (i′) in the definition of a metric space in Chapter 3, §11.
(3) Change hint to Problem 18(iv) in Chapter 3, §12 from

> [Hint: Consider $G_p(\frac{1}{2})$ in a *discrete* space; see §11, Example (3).]

to

> [Hint: Consider $G_p(1)$ in a *discrete* space $(S, \rho)$ with more than one point in $S$; see §11, Example (3).]

(4) Change beginning of Chapter 3, §12, Problem 18(v) from "In $E^n$, if $(a, b) \neq \emptyset$, ..." to "In $E^n$, if $(\bar{a}, \bar{b}) \neq \emptyset$, ..."

Thanks to the students in Math 440, Fall 2014, for their comments.

Version of July 6, 2011

(1) Chapter 1, §§4–7, just before the examples, change

> *it suffices to specify its domain* $D_f$ *and the function value* $f(x)$ *for each* $s \in D_f$

to

> *it suffices to specify its domain* $D_f$ *and the function value* $f(x)$ *for each* $x \in D_f$.

(2) Chapter 2, §§5–6, Problem 7, change

> Hence
> $$\sum_{k=0}^{n} ar^k = a\,\frac{1-r^{n+1}}{1-r}$$

to

> Hence for $r \neq 1$
> $$\sum_{k=0}^{n} ar^k = a\,\frac{1-r^{n+1}}{1-r}$$

(3) Chapter 2, §§5–6, Problem 8, add at the end

> What value must $0^0$ take for (ii) to hold for all $a$ and $b$?

(4) Chapter 2, §13, Problem 7, add the hint:

> [Hint: Prove the first inequality and then use that and Problem 5(i) for the others.]

(5) Chapter 3, §§4–6, Problem 3, change "($\vec{u} = \bar{b} - \bar{a}$)" to "($\vec{u} = \bar{b} - \bar{a} \neq \vec{0}$)".

(6) Chapter 3, §17, Definition 2, change

> A metric space (or subspace) $(S,\,\rho)$ is said to be *complete* if every Cauchy sequence in $S$ converges to some point $p$ *in* $S$.
>
> Similarly, a *set* $A \subseteq (S,\,\rho)$ is called *complete* iff each Cauchy sequence $\{x_m\} \subseteq A$ converges to some point $p$ *in* $A$, i.e., iff $(A,\,\rho)$ is complete as a subspace.

to

> A metric space (or subspace) $(S,\,\rho)$ is said to be *complete* iff every Cauchy sequence in $S$ converges to some point $p$ *in* $S$.
>
> Similarly, a *set* $A \subseteq (S,\,\rho)$ is called *complete* iff each Cauchy sequence $\{x_m\} \subseteq A$ converges to some point $p$ *in* $A$, i.e., iff $(A,\,\rho)$ is complete as a metric subspace of $(S,\rho)$.

(7) Chapter 4, §2, Example (A), change

$$\lim_{n\to\infty}\Big(1+\frac{1}{n}\Big)^{n+1} = \lim_{n\to\infty}\Big(1+\frac{1}{n}\Big)\Big(1+\frac{1}{n}\Big)^{n} = 1\cdot\lim\Big(1+\frac{1}{n}\Big)^{n} = 1\cdot e = e,$$

to

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n}\right)^{n} = 1\cdot \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = 1\cdot e = e,$$

(8) Chapter 4, §3, Problem 11(e), change "$f(0,\,y) = 1$" to "$f(0,\,y) = 0$".

(9) Chapter 4, §8, Problem 12, change the Hint from

> [Hint: Verify that
>
> $$\max(a,\,b) = \frac{1}{2}(a+b+|b-a|) \text{ and } \min(a,\,b) = \frac{1}{2}(a+b-|b-a|)$$
>
> and use Theorem 1 of §3.]

to

> [Hint: After proving the first statements, verify that
>
> $$\max(a,\,b) = \frac{1}{2}(a+b+|b-a|) \text{ and } \min(a,\,b) = \frac{1}{2}(a+b-|b-a|)$$
>
> and use Problem 9 and Example (b).]

(10) Chapter 4, §12, beginning of Theorem 4, change

> Let
> $$f = \sum_{m=1}^{\infty} f_m \text{ on } B.^3$$

to

> Let
> $$f = \sum_{m=1}^{\infty} f_m \text{ (pointwise) on } B.^3$$

and change

> Then
> $$f = \sum_{n=1}^{\infty} g_n \text{ on } B \text{ as well;}$$

to

Then

$$f = \sum_{n=1}^{\infty} g_n \text{ (pointwise) on } B \text{ as well;}$$

(11) Chapter 4, §12, proof of Theorem 4, change

Hence $s_m \to f$ *implies* $s'_n \to f$; i.e.,

$$f = \sum_{n=1}^{\infty} g_n.$$

to

Hence $s_m \to f$ (pointwise) *implies* $s'_n \to f$ (pointwise); i.e.,

$$f = \sum_{n=1}^{\infty} g_n \text{ (pointwise).}$$

(12) Chapter 4, §13, statement of Theorem 2(ii), change

$$\sum_{n=1}^{\infty} |f_m| = +\infty$$

to

$$\sum_{m=1}^{\infty} |f_m| = +\infty$$

(13) Chapter 4, §13, Example (c), change

$$\sum a_n = 2^{-1} + 3^{-1} + 2^{-2} + 3^{-2} + \cdots .$$

to

$$\sum a_n = \frac{1}{2^1} + \frac{1}{3^1} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots .$$

Thanks to the students in MA504, Spring 2011, for some of these changes.

Version of May 24, 2010

(1) In the Preface, change "upalatable to undergraduates" to "unpalatable to undergraduates."
(2) Chapter 2, §§8–9, statement of Theorem 1, change "a infimum" to "an infimum".
(3) Chapter 3, §9, move the text of the footnote to first displayed equation to the page with the displayed equation.
(4) Chapter 3, §15, in the hint to Problem 26, change

    Combining this with (i), we have, for $K' = \max(k',\, k'')$,

    to

    Combining this with (i), we have, for $K' = \max(k, k',\, k'')$,

(5) Chapter 4, §3, beginning of Part II, change

    **II.** If the range space of $f$ is $E^n$ ($^*$or $C^n$), then each function value $f(x)$ is a *vector* in that space; thus it has $n$ real (respectively, complex) components,

    to

    **II.** If the range space of $f$ is $E^n$ ($^*$or $C^n$), then each function value $f(x)$ is a *vector* in that space; thus it has $n$ real ($^*$respectively, complex) components,

(6) Chapter 4, §5, Problem 6, change

$$G_1 = \left(\frac{1}{3}, \frac{2}{3}\right),\ G_{21} = \left(\frac{1}{9}, \frac{2}{9}\right),\ G_{22} = \left(\frac{7}{9}, \frac{8}{9}\right),\ \text{and so on;}$$

    to

$$G_{11} = \left(\frac{1}{3}, \frac{2}{3}\right),\ G_{21} = \left(\frac{1}{9}, \frac{2}{9}\right),\ G_{22} = \left(\frac{7}{9}, \frac{8}{9}\right),\ \text{and so on;}$$

(7) Chapter 4, §9, Problem 4, change

    For functions on $B = [\bar{a},\, \bar{b}] \subset E^1$, Theorem 1 can be proved thusly:

    to

    For functions on $B = [a,\, b] \subset E^1$, Theorem 1 can be proved thusly:

(8) Chapter 4, §10, statement of Theorem 4, change

> Every open connected set $A$ in $E^n$ (or in another normed space) is also arcwise connected and even polygon connected.

to

> Every open connected set $A$ in $E^n$ (*or in another normed space) is also arcwise connected and even polygon connected.

(9) Chapter 4, §12, footnote to Example (a), change "$f^n$ is a monotone function on $C$" to "$f_n$ is a monotone function on $C$."

(10) Chapter 4, §13, hint to Problem 10, change "Use Problem 9' " to "Use Problem $9'$."

(11) Chapter 5, §1, Example (a), change "Then $n$ is continuous on $E^1$" to "Then $f$ is continuous on $E^1$".

(12) Chapter 5, §2, Note 3, change

> Thus Corollary 2 states that *the secant is parallel to the tangent at some intermediate point* $q$;

to

> Thus Corollary 3 states that *the secant is parallel to the tangent at some intermediate point* $q$;

(13) Chapter 5, §5, Note 2, change "Thus, keeping a fixed" to "Thus, keeping $a$ fixed."

(14) Chapter 5, §6, proof of Theorem 1, change

$$-h(t) = \frac{1}{n!}\int_t^a f^{(n+1)}(t)(a-t)^n\,dt \quad \text{on } I$$

to

$$-h(t) = \frac{1}{n!}\int_t^a f^{(n+1)}(s)(a-s)^n\,ds \quad \text{on } I$$

Also, change "As $x=a$, (3') is proved" to "As $x=a$, $(3')$ is proved."

(15) Chapter 5, §6, hint to Problem 8, change "If $0 \le x \le 1$, use (5')" to "If $0 \le x \le 1$, use $(5')$". Also, change "(b) If $-1 \le x \le 0$" to "(b) If $-1 \le x < 0$".

(16) Chapter 5, §6, change hint to Problem 10 from

> [Hint: Use formulas (3) and $(5')$ with $p=a$ and $n=2$, noting that $f'$ *decreases* on $(a,\,b)$ (why?), whence
>
> $$\frac{f(b)-f(a)}{b-a} = f'(q_0) > f'(a)$$
>
> for some $q_0 \in (a,\,b)$. (Explain!)]

to

[Hint: This formula is equivalent to

$$\frac{f(x_0)-f(a)}{x_0-a} > \frac{f(b)-f(a)}{b-a},$$

i.e., the average of $f'$ on $[a, x_0]$ is strictly greater than the average of $f'$ on $[a, b]$, which follows because $f'$ *decreases* on $(a, b)$. (Explain!)]

(17) Chapter 5, §7, proof of Theorem 2, change

$$\begin{aligned}|\Delta_i hf| &= |h(t_i)f(t_i) - h(t_{i-1})f(t_{i-1})| \\ &\le |h(t_i)f(t_i) - h(t_{i-1})f(t_i)| + |h(t_{i-1})f(t_i) - h(t_i)f(t_i)| \\ &= |f(t_i)||\Delta_i h| + |h(t_{i-1})||\Delta_i f| \\ &\le r|\Delta_i h| + s|\Delta_i f|.\end{aligned}$$

to

$$\begin{aligned}|\Delta_i hf| &= |h(t_i)f(t_i) - h(t_{i-1})f(t_{i-1})| \\ &\le |h(t_i)f(t_i) - h(t_{i-1})f(t_i)| + |h(t_{i-1})f(t_i) - h(t_{i-1})f(t_{i-1})| \\ &= |f(t_i)||\Delta_i h| + |h(t_{i-1})||\Delta_i f| \\ &\le r|\Delta_i h| + s|\Delta_i f|.\end{aligned}$$

(18) Chapter 5, §7, Theorem 4, change

A function $f\colon E^1 \to E^n$ $(C^n)$ is of bounded variation on $I = [a,\, b]$ iff all of its components $(f_1,\, f_2,\, \ldots,\, f_n)$ are.

to

A function $f\colon E^1 \to E^n$ $({}^*C^n)$ is of bounded variation on $I = [a,\, b]$ iff all of its components $(f_1,\, f_2,\, \ldots,\, f_n)$ are.

(19) Chapter 5, §8, proof of Theorem 3, displayed formula (3), change

$$\left|\frac{\Delta f}{\Delta x}\right| \le \frac{\Delta v_f}{\Delta x} \le \sup_{[p,x]} |f'(p)| < +\infty \tag{3}$$

to

$$\left|\frac{\Delta f}{\Delta x}\right| \le \frac{\Delta v_f}{\Delta x} \le \sup_{[p,x]} |f'| < +\infty \tag{3}$$

(20) Chapter 5, §9, proof of Theorem 3, change

> Finally, to prove (iv), we apply (i)–(iii) to the consecutive derivatives of $f$ and obtain
>
> $$f^{(k)}(x) = \sum_{n=k}^{\infty} n(n-1)\cdots(n-k-1)a_n(x-p)^{n-k}$$
>
> for each $x \in I$ and $k \in N$.

to

> Finally, to prove (iv), we apply (i)–(iii) to the consecutive derivatives of $f$ and obtain
>
> $$f^{(k)}(x) = \sum_{n=k}^{\infty} n(n-1)\cdots(n-k+1)a_n(x-p)^{n-k}$$
>
> for each $x \in I$ and $k \in N$.

(21) Chapter 5, §10, proof of Lemma 2, change

> If $x \in (p_1, p_1 + \delta_1)$, then
>
> $$g(x) = f(p^+).$$

to

> If $x \in (p_1, p_1 + \delta_1)$, then
>
> $$g(x) = f(p_1^+).$$

(22) Chapter 5, §7, end of hint to Problem 6, change "What if $g(c) = f$, $g(d) = a$?" to "What if $g(c) = b$, $g(d) = a$?"

(23) Chapter 5, §8, Problem 5(ii), change "$F$ and and $f$ describe the same simple arc" to "$F$ and $f$ describe the same simple arc."

(24) Chapter 5, §10, proof of Theorem 2, change "$F = d$ on $[b, d]$" to "$F = d$ on $[d, b]$."

(25) Chapter 5, §10, proof of Theorem 4, just after "Combining all, ..." replace

$$\begin{aligned}\int fg &= f(b)G(b) - \int_a^b f'G(q)\\ &= f(b)G(b) - f(b)G(q) + f(a)G(q)\\ &= f(b)\int_q^b g + f(a)\int_a^q g. \ \square\end{aligned}$$

by

$$\begin{aligned}\int fg &= f(b)G(b) - \int_a^b Gf'\\ &= f(b)G(b) - f(b)G(q) + f(a)G(q)\\ &= f(b)\int_q^b g + f(a)\int_a^q g. \ \square\end{aligned}$$

(26) Chapter 5, §10, Problem 5, change the displayed equation in the hint from

$$f(x) = x \cdot \sin\frac{1}{x},\ g(x) = \frac{x}{|x|},\ \text{and } f(0) = g(0) = \ \text{with } I = [0,\,1].$$

to

$$f(x) = x \cdot \sin\frac{1}{x},\ g(x) = \frac{x}{|x|},\ \text{and } f(0) = g(0) = 0 \text{ with } I = [0,\,1].$$

(27) Chapter 10, §11, last displayed equation of the proof of Theorem 4, change

$$s'_-(q) = \lim_{x\to q} c(x) = c(q), \quad \text{as required.}$$

to

$$s'_-(q) = \lim_{x\to q^-} c(x) = c(q), \quad \text{as required.}$$

Thanks to H. Khass for these many corrections.

Version of March 10, 2009

(1) Chapter 2, §§1–4, Axiom III of the real numbers, change

$$(\forall\, x,\, y,\, z \in E^1) \quad (x+y)+z = x+(y+x) \text{ and } (xy)z = x(yz).$$

to

$$(\forall\, x,\, y,\, z \in E^1) \quad (x+y)+z = x+(y+z) \text{ and } (xy)z = x(yz).$$

(2) Chapter 2, §§5–6, rewrite Problem 8 to be

8. For $n > 0$ define

$$\binom{n}{k} = \begin{cases} \dfrac{n!}{k!\,(n-k)!}, & 0 \le k \le n, \\ 0, & \text{otherwise.} \end{cases}$$

Verify *Pascal's law*,

$$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}.$$

Then prove by induction on $n$ that

(i) $(\forall\, k \mid 0 \le k \le n)\ \binom{n}{k} \in N$; and

(ii) for any field elements $a$ and $b$,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}, \quad n \in N \text{ (the \textit{binomial theorem})}.$$

(3) Chapter 2, §13, last displayed formula of the proof of Theorem 1, change

$$q_n = \sup A_n \le a.$$

to

$$(\forall\, n > n_0) \quad q_n = \sup A_n \le a.$$

(4) Chapter 3, §9, Example (d), change

with each map $f\colon A \to W$ treated as a single "vector" in $W$

to

with each map $f\colon A \to V$ treated as a single "vector" in $W$.

(5) Chapter 3, §13, Problem 9, change

In case (i), show that globes of radius $\le 1$ are the same under $\rho$ and $\rho'$. In case (ii), prove that any $G_p(\epsilon)$ in $(S, \rho)$ is also a globe in $(S, \rho')$ of radius

$$\frac{\epsilon}{1+\epsilon},$$

and any globe of radius $< 1$ in $(S, \rho')$ is also a globe in $(S, \rho)$. (Find the converse formula as well!)

to

> In case (i), show that globes $G_p(\epsilon)$ of radius $\epsilon \leq 1$ are the same under $\rho$ and $\rho'$. In case (ii), prove that any $G_p(\epsilon)$ in $(S,\, \rho)$ is also a globe $G_p(\epsilon')$ in $(S,\, \rho')$ of radius
>
> $$\epsilon' = \frac{\epsilon}{1+\epsilon},$$
>
> and any globe of radius $\epsilon' < 1$ in $(S,\, \rho')$ is also a globe in $(S,\, \rho)$. (Find the converse formula for $\epsilon$ as well!)

(6) Chapter 3, §11, hint to Problem 10, change

$$\rho'_{13} \leq \rho'_{12} + \rho'_{23} \text{ and } \rho''_{13} \leq \rho''_{22} + \rho''_{23}.$$

to

$$\rho'_{13} \leq \rho'_{12} + \rho'_{23} \text{ and } \rho''_{13} \leq \rho''_{12} + \rho''_{23}.$$

(7) Chapter 3, §12, Problem 3, change

> Prove that if $\bar{p} \in G_{\bar{q}}(r)$ in $E^n$, then $G_{\bar{q}}(r)$ contains a cube $[\bar{c},\, \bar{d}]$ with center $\bar{p}$

to

> Prove that if $\bar{p} \in G_{\bar{q}}(r)$ in $E^n$, then $G_{\bar{q}}(r)$ contains a cube $[\bar{c},\, \bar{d}]$ with $\bar{c} \neq \bar{d}$ and with center $\bar{p}$.

(8) Chapter 3, §14, Problem 10, change

$$\left\{1,\, 1,\, \frac{1}{2},\, 1\frac{1}{2},\, \ldots,\, \frac{1}{n},\, 1+\frac{1}{n},\, \ldots\right\}$$

to

$$\left\{1,\, 2,\, \frac{1}{2},\, 1\frac{1}{2},\, \ldots,\, \frac{1}{n},\, 1+\frac{1}{n},\, \ldots\right\}$$

(9) Chapter 3, §15, hint to Problem 38, change

> [Hint: Take limits of both sides in $a_{n+1} = \frac{1}{2}(a_n + b_n)$ to get $p = \frac{1}{2}(p+q)$.]

to

> [Hint: Take limits of both sides in $b_{n+1} = \frac{1}{2}(a_n + b_n)$ to get $q = \frac{1}{2}(p+q)$.]

(10) Chapter 3, §16, Problem 15, change "Find such *quantifier formulas* for $p \in A$" to "Find such *quantifier formulas* for $p \in \overline{A}$".

Thanks to the students of Math 504, Spring 2009, at Purdue for their comments.

Version of November 19, 2007

(1) Chapter 3, §15, Problem 25. In part (e), change

$$x_n = \sqrt{\sum_{k=1}^{m} a_k^n}\,, \text{ with } a_k > 0$$

to

$$x_n = \sqrt[n]{\sum_{k=1}^{m} a_k^n}\,, \text{ with } a_k > 0.$$

In the hint to part (a), change

$$0 < x_n < n^q\left[\left(1+\frac{1}{n}\right)^q - 1\right] < n^q$$

to

$$0 < x_n = n^q\left[\left(1+\frac{1}{n}\right)^q - 1\right] < n^q.$$

Thanks to Jonathan Ferron for his comments.

Version of May 14, 2007.

(1) Chapter 4, §5, statement of Theorem 1. Change

$$f(p^+) = \inf_{a<x<b} f(x) \text{ for } p \in [a,\, b)$$

to

$$f(p^+) = \inf_{p<x<b} f(x) \text{ for } p \in [a,\, b).$$

(2) Chapter 4, §13, before Theorem 1. Change "We now consider absolute convergence for *complete* range spaces" to "For the rest of this section we consider only *complete* range spaces."

(3) Chapter 4, §13, proof of Theorem 5. Change

we similarly obtain $|a_{n+1}| < |a_n| r$; hence by induction,

$$(\forall\, n \in N) \quad |a_n| \le |a_1| r^{n-1}.$$

to

we similarly obtain $(\exists\, m)\ (\forall\, n \ge m)\ |a_{n+1}| < |a_n| r$; hence by induction,

$$(\forall\, n \ge m) \quad |a_n| \le |a_m| r^{n-m}.$$

(4) Chapter 4, §13, Example (C). Change

$$\overline{\lim}\ \sqrt[n]{a_n} = \lim\ \sqrt[2n]{2^{-n}} = \frac{1}{\sqrt{2}} < 1$$

to

$$\overline{\lim}\ \sqrt[n]{a_n} = \lim\ \sqrt[2n-1]{2^{-n}} = \frac{1}{\sqrt{2}} < 1.$$

(5) Chapter 5, §1, Problem 7(i). Add the hint "For the converse, start with Problem 14(iii) of Chapter 4, §2."

(6) Chapter 5, §2, proof of Theorem 1. Change the second paragraph and the beginning of the third paragraph to

> Thus $M = f(a)$ or $M = f(b)$; for the moment we assume $M = f(b)$ and $m = f(a)$. We must have $m < M$, for $m = M$ would make $f$ *constant* on $[a, b]$, implying $f' = 0$. Thus $m = f(a) < f(b) = M$.
>
> Now let $a \le x < y \le b$. Applying the previous argument to each of the intervals $[a, x]$, $[a, y]$, $[x, y]$, and $[x, b]$ (now using that $m = f(a) < f(b) = M$), we find that
>
> $$f(a) \le f(x) < f(y) \le f(b). \quad \text{(Why?)}$$

Thanks to the students of the Spring 2007 section of MA 504 for their comments.

Version of April 22, 2007

(1) Chapter 3, §14, displayed equation just after Corollary 1. Change

$$G_p(\epsilon) \cap G_p(\epsilon)$$

to

$$G_p(\epsilon) \cap G_q(\epsilon).$$

(2) Chapter 4, §1, statement of Corollary 1. Change "$\lim\limits_{x\to\rho} f(x)$" to "$\lim\limits_{x\to p} f(x)$".

(3) Chapter 4, §2, statement of Theorem 1. Change

$$f\colon A \to (T, \rho), \text{ with } A \subseteq (S, \rho)$$

to

$$f\colon A \to (T, \rho'), \text{ with } A \subseteq (S, \rho).$$

(4) Chapter 4, §4, Note 1. Change

(i) $\dfrac{(\pm\infty)}{0^+} = \pm\infty$, $\dfrac{(\pm\infty)}{0^-} = \mp\infty$. If $q > 0$, then $\dfrac{q}{0^+} = +\infty$ and $\dfrac{q}{0^-} = -\infty$.

(ii) $\dfrac{\infty}{0} = \infty$.
(iv) $\dfrac{q}{\infty} = 0$.
(v) If $0 < |q| < 1$, then $q^{-\infty} = \infty$ and $q^{+\infty} = 0$.
(vi) If $|q| > 1$, then $q^{+\infty} = \infty$ and $q^{-\infty} = 0$.

to

(i) $\dfrac{(\pm\infty)}{0^+} = \pm\infty$, $\dfrac{(\pm\infty)}{0^-} = \mp\infty$.
(ii) If $q > 0$, then $\dfrac{q}{0^+} = +\infty$ and $\dfrac{q}{0^-} = -\infty$.
(iii) $\dfrac{\infty}{0} = \infty$.
(iv) $\dfrac{q}{\infty} = 0$.

(5) Chapter 4, §5, statement of Theorem 3. Change "If $f\colon A \to E^*$ is monotone on a finite or infinite interval $(a, b)$ containing $A$ ..." to " If $f\colon A \to E^*$ is monotone on a finite or infinite interval $(a, b)$ contained in $A$ ..."

(6) Chapter 4, §6, proof of Theorem 5. Change "Therefore, by the definition of completeness (Chapter 3, §17), $\{x_m\}$ has a limit $p \in E_1$" to "Therefore, by the definition of completeness (Chapter 3, §17), $\{x_m\}$ has a limit $p \in F_1$."

(7) Chapter 4, §8, statement of Theorem 1. Change "a function $f\colon A \to (T, \rho)$" to "a function $f\colon A \to (T, \rho')$" and "Briefly, any continuous image $f[B]$ of a compact set $B$ is compact itself" to "Briefly, the continuous image of a compact set is compact."

(8) Chapter 4, §8, Problem 12. Change

$$(f \vee g)(x) = \max((f(x), f(g))$$

to

$$(f \vee g)(x) = \max((f(x), g(x)).$$

(9) Chapter 4, §9, proof of Theorem 1. Add a paragraph break before "We denote this particular closed segment by $L[\bar{p}_1, \bar{q}_1]$, ..."

(10) Chapter 4, §11, Definition 2, change "A function $f\colon (X \times Y) \to (T, \rho)$ ..." to "A function $f\colon (X \times Y) \to (T, \rho')$ ..."

(11) Chapter 4, §11, statement of Theorem 2. Change "Let $(T, \rho)$ be complete" to "Let $(T, \rho')$ be complete."

(12) Chapter 4, §12, Example (a), footnote 2. Keep footnote on page with footnote mark.

(13) Chapter 4, §12, Problem 3(iv). Change "$f_n(x) = \dfrac{nx^2}{1 + nx}$" to "$f_n(x) = \dfrac{x}{1 + nx}$."

(14) Chapter 4, §13, statement of Theorem 1, footnote 4. Keep footnote on page with footnote mark.

Thanks to the students of the Spring 2007 section of MA 504 for their comments.

Version of February 19, 2007

(1) Chapter 2, §§8–9, end of proof of Theorem 1. Change

$$p = \operatorname{glb} A = \inf B. \quad \square$$

to

$$p = \operatorname{glb} A = \inf A. \quad \square$$

(2) Chapter 2, §13, after Corollary 1. Change

$$\overline{L} \geq p_n = \inf A_n \geq \inf A_1 = \inf_n x_n \text{ and}$$
$$\overline{L} \leq q_n = \sup A_n \leq \sup A_1 = \sup_n x_n,$$

to

$$\underline{L} \geq p_n = \inf A_n \geq \inf A_1 = \inf_n x_n \text{ and}$$
$$\overline{L} \leq q_n = \sup A_n \leq \sup A_1 = \sup_n x_n,$$

(3) Chapter 2, §13, proof of Theorem 2. Change "Now, exactly as in the proof of Theorem 2, one *excludes*

$$q \neq \underline{\lim}\, x_n \text{ and } q \neq \underline{\lim}\, x_n.\text{"}$$

to "Now, exactly as in the proof of Theorem 2, one *excludes*

$$q \neq \underline{\lim}\, x_n \text{ and } q \neq \overline{\lim}\, x_n.\text{"}$$

(4) Chapter 3, §8, just before Definition 1. Change "In particular, if $z = (x, y)$ and $z'(x', y')$, we have ..." to "In particular, if $z = (x, y)$ and $z' = (x', y')$, we have ...".

(5) Chapter 3, §8, Problem 7. Change "Prove that if $z = z'z''$, then ..." to "Prove that $z = z'z''$ if ...".

(6) Chapter 3, §12, hint to Problem 11. Change

$$\bigcup_{n=1}^{\infty} \left[\frac{1}{n}, 1 - \frac{1}{n}\right] = (0, 1).$$

to

$$\bigcup_{n=2}^{\infty} \left[\frac{1}{n}, 1 - \frac{1}{n}\right] = (0, 1).$$

(7) Chapter 3, §13, Problem 9(d). Change

$$\frac{n(n-1)}{(n-2)^2}$$

to

$$\frac{n(n-1)}{(n+2)^2}.$$

(8) Chapter 3, §15, Theorem 1, statement of part (iii). Change

$$\frac{x_m}{a_m} \to \frac{q}{a} \text{ if } a \neq 0.$$

to

$$\frac{x_m}{a_m} \to \frac{q}{a} \text{ if } a \neq 0 \text{ and for all } m \geq 1,\ a_m \neq 0.$$

(9) Chapter 3, §15, Example. Change

$$\begin{aligned} 2 < x_n &< 2 + \frac{1}{2!} + \cdots + \frac{1}{n!} \\ &\leq 2 + \frac{1}{2} + \cdots + \frac{1}{2^n} = 2 + \frac{1}{2}\,\frac{1 - \left(\frac{1}{2}\right)^{n+1}}{\frac{1}{2}} < 2 + 1 = 3. \end{aligned}$$

to

$$\begin{aligned} 2 < x_n < 2 + \frac{1}{2!} + \cdots + \frac{1}{n!} &\leq 2 + \frac{1}{2} + \cdots + \frac{1}{2^{n-1}} \\ = 2 + \frac{1}{2}\left(1 + \cdots + \frac{1}{2^{n-2}}\right) &= 2 + \frac{1}{2}\,\frac{1 - \left(\frac{1}{2}\right)^{n-1}}{\frac{1}{2}} < 2 + 1 = 3. \end{aligned}$$

(10) Chapter 3, §15, Corollary 1. Change "Suppose $\lim s_m = p$" to "Suppose $\lim x_m = p$".

Thanks to the students of the Spring 2007 section of MA 504 for their comments.

Version of January 5, 2007

(1) Chapter 5, §6, Proof of Theorem 1′. Change references to Cauchy's law of the mean to "Theorem 2 of §2" from "Theorem 3 of §2".

(2) Chapter 5, §6, Example (a). Change

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \frac{e^{\theta_n x}}{(n+1)!}, \quad 0 < \theta_n < 1. \tag{8}$$

to

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \frac{e^{\theta_n x}}{(n+1)!}\, x^{n+1}, \quad 0 < \theta_n < 1. \tag{8}$$

Indicate that the remainder $R_n$ is positive when $x > 0$.

(3) Chapter 5, §7, end of page 300. Change

$$|q_i - q| + |q - q_{i-1}| \geq |q_i - q_i| \quad \text{(triangle law)}$$

to

$$|q_i - q| + |q - q_{i-1}| \geq |q_i - q_{i-1}| \quad \text{(triangle law).}$$

Thanks to Daniele Pala for the changes.

Version of July 26, 2006

(1) Chapter 4, §12, Problem 14. Part (iii), change "$\lim_{x\to-\infty} \frac{a^x}{x^q} = 0$" to "$\lim_{x\to\infty} \frac{a^{-x}}{x^q} = 0$". Part (iv), change "$\lim_{x\to-\infty} \frac{a^x}{x^q} = +\infty$" to "$\lim_{x\to\infty} \frac{a^{-x}}{x^q} = +\infty$".

(2) Chapter 4, §13, proof of Theorem 5. Refer to Problem 17, not Problem 18, in Chapter 4, §12.

(3) Chapter 5, §1, Example (b). Change "$f'(x) = \sin x - i \cdot \cos x$" to "$f'(x) = -\sin x + i \cdot \cos x$".

(4) Chapter 5, §4, proof of Lemma 1. Change "$|f(x) - f(x)| \leq \ldots$" to "$|f(x) - f(r)| \leq \ldots$".

(5) Chapter 5, §5, Note 2. Change "$G(a) = F(a) - f(a)$" to "$G(a) = F(a) - F(a)$".

(6) Chapter 5, §5, proof of Corollary 1. Change "as are $f$ and $g$" to "as are $F$ and $G$."

Thanks to Daniele Pala for the changes.

Version of July 25, 2006:

(1) Chapter 3, §12, Problem 11, in the hint, change

$$\bigcup_{n=1}^{\infty} \left(-\frac{1}{n}, \frac{1}{n}\right) = \{0\}$$

to

$$\bigcap_{n=1}^{\infty} \left(-\frac{1}{n}, \frac{1}{n}\right) = \{0\}$$